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Substituting the values obtained in MacLaurin's Formula, we have :

$$\tan x = x + \frac{2x^3}{3\,!} + \frac{16x^5}{5\,!} + \frac{272x^7}{7\,!} + \frac{7936x^9}{9\,!} + \frac{353792x^{11}}{11\,!} + \frac{22368256x^{13}}{13\,!} + \text{etc.}$$

$$= x + \frac{x^3}{3} + \frac{2x^5}{3\times5} + \frac{17x^7}{3^2\times5\times7} + \frac{62x^9}{3^2\times5\times7\times9} + \frac{1382x^{11}}{3^2\times5^2\times7\times9\times11}$$

$$+ \frac{21844x^{13}}{3^3\times5^2\times7\times9\times11\times13}.$$

II. Solution by H. C. WHITAKER, A. M., Ph. D., Professor of Mathematics, Manual Training School, Philadelphia, Pa., and J. SCHEFFER, A. M., Hagerstown, Md.

The general term of the series is

$$\frac{2^{2n}(2^{2n}-1)}{2n\,!}Bx^{2n-1},$$

where B (Bernoulli) stands for the coefficient of x^n in the expansion of

$$\frac{x}{e^x-1} = \frac{x}{\frac{x}{1}+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+\text{etc.}}$$

120. Proposed by JOSIAH H. DRUMMOND, LL. D., Portland, Me.

A hollow sphere has within it a solid sphere; a quantity of water equal to $1/m$ of the capacity of the hollow sphere is poured in and just covers the solid sphere. Prove that there are two solid spheres, either of which answers the conditions; also find the maximum value $1/m$, beyond which the question is not possible.

I. Solution by J. A. COLSON, Searsport, Me.

The volume of the inner sphere (radius r)+the volume of the water=the volume of a segment whose radius is R and whose height is $2r$; that is

$$\tfrac{4}{3}\pi r^3 + \frac{4}{3m}\pi R^3 = \tfrac{1}{3}\pi(2r)^2(3R-2r),$$

which reduces to $r^3 - Rr^2 + R^3/3m = 0$....(1).

Hence, by Cardan's Method one value of r is

$$r_1 = \tfrac{1}{3}R + \sqrt[3]{\left[\tfrac{1}{2}\left[-\frac{R^3}{27}\left(\frac{9}{m}-2\right)+\frac{R^3}{9}\sqrt{\left(\frac{9}{m^2}-\frac{4}{m}\right)}\right]\right]}$$

$$+\sqrt[3]{\left[\tfrac{1}{2}\left[-\frac{R^3}{27}\left(\frac{9}{m}-2\right)-\frac{R^3}{9}\sqrt{\left(\frac{9}{m^2}-\frac{4}{m}\right)}\right]\right]}\ldots\ldots(2).$$

Now (1) will have either one or three real roots. If three, two of them by Descarte's Rule of Signs will be positive and one negative. In the former case the quantity $9/m^2-4/m$ under the radical sign in (2) will be positive. In the latter case it will be negative. Consequently the point of change from the reducible to the irreducible case of Cardan will be when $9/m^2-4/m=0$, which gives $1/m=\frac{4}{9}$. Substituting this value in (1) we have $r^3-Rr^2+4R^3/27=0$..(3), whose roots are $r_1=-R/3$, and $r_2=r_3=\frac{2}{3}R$. The positive roots being equal shows that $\frac{4}{9}$ is the maximum value of $1/m$ which will give positive roots.

II. Solution by the PROPOSER.

Let a be the radius of the hollow sphere, and x the diameter of the solid sphere, and, therefore, the weight of the water. Then the center of the hollow sphere is $x-a$ or $a-x$ from the surface of the water; and we find that $\sqrt{(2ax-x^2)}$=radius of surface of the water, and as the amount of water=one half the cylinder having area of surface of water for base and height of water for altitude it is equal to $\frac{1}{2}\pi x^2(2a-x)=(4\pi na^3)/3$, (using n instead of $1/m$). Reducing we have $x^3-2ax^2=(-8na^3)/3$. This being a cubic equation, it has three roots; and as their product is negative, one, or all of them must be negative; but as their sum is positive at least one of them must be positive; and taking both of them together, there must be two positive roots, and therefore x has two values, each of which answers the conditions. From our equation we have $n=3(2ax^2-x^3)/8a^3$. Differentiating and reducing we find $x=4a/3$, and substituting this in the expression for the value of n, we have $n=1/m=\frac{4}{9}$. When $1/m=\frac{4}{9}$, $x=4a/3$, and the equation has two equal positive roots—*practically* but not *mathematically*, an exception to the proposition that x has two values, each of which satisfies the conditions.

Also solved by *H. C. WHITAKER* and *G. B. M. ZERR*.

121. Proposed by ELMER SCHUYLER, B. Sc., Professor of German and Mathematics, Boys' High School, Reading, Pa.

Solve $(x^5+y^5+z^5)^3+(x+y)^2=31$.

$(x^5+y^5+z^5)^3+(x+y+z)^3=729$.

$(x+y)^2+(x+y+z)^3=31$.

Solution by W. F. BUCK, Instructor in the Science Department, Leominster High School, Leominster, Mass.

Let $x^5+y^5+z^5=r$....(4).

$x+y=s$....(5).

$x+y+z=t$....(6).

Then from the original equations,

$$r^3+s^2=31,\quad r^3+t^3=729,\quad s^2+t^3=31,$$

which easily give

$$r=\sqrt[3]{\tfrac{729}{2}},\quad s=\sqrt{-\tfrac{667}{2}},\quad t=\sqrt[3]{\tfrac{729}{2}}\text{....(7)}.$$